**20.** Show that $f(x) = 2x + \cot^{-1} x + \log\,(\sqrt{1+x^2} - x)$ is increasing in $R$.

**Sol.** We have, $f(x) = 2x + \cot^{-1} x + \log\,(\sqrt{1+x^2} - x)$

$$\therefore \qquad f'(x) = 2 - \frac{1}{1+x^2} + \frac{1}{\sqrt{1+x^2} - 1}\left(\frac{x}{\sqrt{1+x^2}} - 1\right)$$

$$= \frac{1}{1+x^2} + \frac{1}{\sqrt{1+x^2} - x} \cdot \frac{x - \sqrt{1+x^2}}{\sqrt{1+x^2}} = 2 - \frac{1}{1+x^2} - \frac{1}{\sqrt{1+x^2}}$$

Now $1 + x^2, \sqrt{1+x^2} \geq 1$ for all real $x$,

$\therefore 2 - \dfrac{1}{1+x^2} - \dfrac{1}{\sqrt{1+x^2}} > 0$ for all real $x$,

$\therefore f'(x) > 0$ for all real $x$.

Thus $f(x)$ is increasing on $R$.