

Example 5 Determine for which values of x , the function $y = x^4 - \frac{4x^3}{3}$ is increasing and for which values, it is decreasing.

Solution $y = x^4 - \frac{4x^3}{3} \quad \Rightarrow \quad \frac{dy}{dx} = 4x^3 - 4x^2 = 4x^2(x - 1)$

Now, $\frac{dy}{dx} = 0 \Rightarrow x = 0, x = 1.$

Since $f'(x) < 0 \quad \forall x \in (-\infty, 0) \cup (0, 1)$ and f is continuous in $(-\infty, 0]$ and $[0, 1]$. Therefore f is decreasing in $(-\infty, 1]$ and f is increasing in $[1, \infty)$.

Note: Here f is strictly decreasing in $(-\infty, 0) \cup (0, 1)$ and is strictly increasing in $(1, \infty)$.