PROBLEM

Let $f'(x) = e^{x^2}$ and $f(0) = 10$. If $A < f(1) < B$ can be concluded from the mean value theorem, then the largest value of $(A - B)$ equals

a. e

b. $1 - e$

c. $e - 1$

d. $1 + e$

SOLUTION

b. Applying LMVT in [0, 1] to the function $y = f(x)$, we get

$$f'(c) = \frac{f(1) - f(0)}{1 - 0}, \text{ for some } c \in (0, 1)$$

or $e^{c^2} = \dfrac{f(1) - f(0)}{1}$

or $f(1) - 10 = e^{c^2}$ for some $c \in (0, 1)$

But $1 < e^{c^2} < e$ in $(0, 1)$

or $1 < f(1) - 10 < e$

or $11 < f(1) < 10 + e$

or $A = 11, B = 10 + e$

or $A - B = 1 - e$