PROBLEM

Let $f(1) = -2$ and $f'(x) \geq 4.2$ for $1 \leq x \leq 6$. The smallest possible value of $f(6)$ is

a. 9

b. 12

c. 15

d. 19

SOLUTION

d. Using Lagrange's mean value theorem, for some $c \in (1, 6)$

$$f'(c) = \frac{f(6) - f(1)}{5} = \frac{f(6) + 2}{5} \geq 4.2$$

or $f(6) + 2 \geq 21$

or $f(6) \geq 19$