

PROBLEM

The value of c in Lagrange's theorem for the function $f(x) = \log \sin x$ in the interval $[\pi/6, 5\pi/6]$ is

a. $\pi/4$

b. $\pi/2$

c. $2\pi/3$

d. none of these

SOLUTION

$$\text{b. } f\left(\frac{5\pi}{6}\right) = \log \sin\left(\frac{5\pi}{6}\right) = \log \sin \frac{\pi}{6} = \log \frac{1}{2} = -\log 2$$

$$f\left(\frac{\pi}{6}\right) = \log \sin \frac{\pi}{6} = -\log 2$$

$$f'(c) = \frac{1}{\sin x} \cos x = \cot x$$

By Lagrange's mean value theorem,

$$\frac{f(5\pi/6) - f(\pi/6)}{(5\pi/6) - (\pi/6)} = \cot c$$

$$\text{or } \cot c = 0 \text{ or } c = \frac{\pi}{2}$$

$$\text{Thus, } c = \frac{\pi}{2} \in (\pi/6, 5\pi/6).$$