PROBLEM

If $2a+3b+6c=0$, then at least one root of the equation $ax^2+bx+c=0$ lies in the interval **[2004|**

(a) $(1,3)$ (b) $(1,2)$ (c) $(2,3)$ (d) $(0,1)$

SOLUTION

Let $f(x)=\frac{ax^3}{3}+\frac{bx^2}{2}+cx \Rightarrow f(0)=0$ and $f(1)$

$$=\frac{a}{3}+\frac{b}{2}+c=\frac{2a+3b+6c}{6}=0$$

Also $f(x)$ is continuous and differentiable in [0, 1] and [0, 1[. So by Rolle's theorem, $f'(x)=0$.

i.e $ax^2+bx+c=0$ has at least one root in [0, 1].