# Subjective Problems

## PROBLEM

Using Rolle's theorem, prove that there is at least one root in $(45^{1/100}, 46)$ of the polynomial

$P(x) = 51x^{101} - 2323(x)^{100} - 45x + 1035.$ ***(2004 - 2 Marks)***

## SOLUTION

We are given,

$$P(x) = 51x^{101} - 2323x^{100} - 45x + 1035$$

To show that at least one root of $P(x)$ lies in $(45^{1/100}, 46)$, using Rolle's theorem, we consider antiderivative of $P(x)$

i.e. $F(x) = \dfrac{x^{102}}{2} - \dfrac{2323x^{101}}{101} - \dfrac{45x^2}{2} + 1035x$

Then being a polynominal function $F(x)$ is continuous and differentiable.

Now, $F(45^{1/100}) = \frac{(45)^{\frac{102}{100}}}{2} - \frac{2323(45)^{\frac{101}{100}}}{101}$

$$- \frac{45.(45)^{\frac{2}{100}}}{2} + 1035(45)^{\frac{1}{100}}$$

$$= \frac{45}{2}(45)^{\frac{2}{100}} - 23 \times 45(45)^{\frac{1}{100}}$$

$$- \frac{45.(45)^{\frac{2}{100}}}{2} + 1035(45)^{\frac{1}{100}} = 0$$

And $F(46) = \frac{(46)^{102}}{2} - \frac{2323(46)^{101}}{101} - \frac{45(46)^2}{2} + 1035(46)$

$= 23\,(46)^{101} - 23\,(46)^{101} - 23 \times 45 \times 46 + 1035 \times 46 = 0$

$\therefore \quad F(45^{\frac{1}{100}}) = F(46) = 0$

$\therefore$ Rolle's theorem is applicable.

Hence, there must exist at least one root of $F'(x) = 0$