PROBLEM

Let $P(x) = a_0 + a_1x^2 + a_2x^4 + \ldots\ldots + a_nx^{2n}$ be a polynomial in a real variable x with

$0 < a_0 < a_1 < a_2 < \ldots\ldots < a_n$. The function $P(x)$ has

(a) neither a maximum nor a minimum ***(1986 - 2 Marks)***

(b) only one maximum

(c) only one minimum

(d) only one maximum and only one minimum

(e) none of these.

SOLUTION

(c) We have $P'(x) = 2a_1x + 4a_2x^3 + \ldots + 2na_nx^{2n-1}$

$$P'(x) = 0 \Rightarrow x = 0$$

$$P''(x) = 2a_1 + 12a_2x^2 + \ldots + 2n(2n-1)a_nx^{2n-2}$$

$$P''(x)|_{x=0} = +ve \text{ as } a_1 > 0$$

$\therefore$ $P(x)$ has only one minimum at $x = 0$.