MCQs with One Correct Answer

PROBLEM

If $a+b+c=0$, then the quadratic equation $3ax^2+2bx+c=0$ has *(1983 - 1 Mark)*

(a) at least one root in [0, 1]
(b) one root in [2, 3] and the other in [–2, –1]
(c) imaginary roots
(d) none of these

SOLUTION

(a) Consider the function $f(x) = ax^3 + bx^2 + cx$ on [0, 1] then being a polynomial. It is continuous on [0, 1], differentiable on (0, 1) and
$f(0)=f(1)=0$ [as given $a + b + c = 0$]

$\therefore$ By Rolle's theorem $\exists\ x \in (0,1)$ such that

$$f'(x) = 0 \Rightarrow 3ax^2 + 2bx + c = 0$$

Thus equation $3ax^2 + 2bx + c = 0$ has at least one root in [0, 1].