76. $f(x) = \sqrt{25 - x^2}$ in $[1, 5]$.

Sol. We have, $f(x) = \sqrt{25 - x^2}$ in $[1, 5]$

Since $25 - x^2$ and square root function are continuous and differentiable in their domain, given function $f(x)$ is also continuous and differentiable.

So, conditions of mean value theorem are satisfied.

Hence, there exists atleast one $c \in (1, 5)$ such that,

$$f'(c) = \frac{f(5) - f(1)}{5 - 1}$$

$$\Rightarrow \quad \frac{-c}{\sqrt{25 - c^2}} = \frac{0 - \sqrt{24}}{4}$$

$$\Rightarrow \quad 16c^2 = 24(25 - c^2)$$

$$\Rightarrow \quad 40c^2 = 600$$

$$\Rightarrow \quad c^2 = 15$$

$$\Rightarrow \quad c = \sqrt{15} \in (1, 5)$$

Hence, mean value theorem has been verified.