74. $f(x) = x^3 - 2x^2 - x + 3$ in $[0, 1]$.

Sol. We have, $f(x) = x^3 - 2x^2 - x + 3$ in $[0, 1]$

Since, $f(x)$ is a polynomial function it is continuous in $[0, 1]$ and differentiable in $(0, 1)$

Thus, conditions of mean value theorem are satisfied.

Hence, there exists a real number $c \in (0, 1)$ such that

$$f'(c) = \frac{f(1) - f(0)}{1 - 0}$$

$$\Rightarrow \quad 3c^2 - 4c - 1 = \frac{[1 - 2 - 1 + 3] - [0 + 3]}{1 - 0}$$

$$\Rightarrow \quad 3c^2 - 4c - 1 = -2$$

$$\Rightarrow \quad 3c^2 - 4c + 1 = 0$$

$$\Rightarrow \quad (3c - 1)(c - 1) = 0$$

$$\Rightarrow \quad c = \frac{1}{3} \in (0, 1)$$

Hence, the mean value theorem has been verified.

75. $f(x) = \sin x - \sin 2x$ in $[0, \pi]$.

Sol. We have, $f(x) = \sin x - \sin 2x$ in $[0, \pi]$

We know that all trigonometric functions are continuous and differentiable in their domain, given function is also continuous and differentiable

So, conditions of mean value theorem are satisfied.

Hence, there exists atleast one $c \in (0, \pi)$ such that,

$$f'(c) = \frac{f(\pi) - f(0)}{\pi - 0}$$

$$\Rightarrow \quad \cos c - 2\cos 2c = \frac{\sin \pi - \sin 2\pi - \sin 0 + \sin 0}{\pi - 0}$$

$$\Rightarrow \quad 2\cos 2c - \cos c = 0$$

$$\Rightarrow \quad 2(2\cos^2 c - 1) - \cos c = 0$$

$$\Rightarrow \quad 4\cos^2 c - \cos c - 2 = 0$$

$$\Rightarrow \quad \cos c = \frac{1 \pm \sqrt{1 + 32}}{8} = \frac{1 \pm \sqrt{33}}{8}$$

$$\Rightarrow \quad c = \cos^{-1}\left(\frac{1 \pm \sqrt{33}}{8}\right) \in (0, \pi)$$

Hence, mean value theorem has been verified.