Direction for Exercises 73 to 76:

Verify Mean Value Theorem for each of the functions given in these exercises.

73. $f(x) = \dfrac{1}{4x-1}$ in $[1, 4]$

Sol. We have, $f(x) = \dfrac{1}{4x-1}$ in $[1, 4]$

Clearly $f(x)$ is continuous in $[1, 4]$

Also $f'(x) = -\dfrac{4}{(4x-1)^2}$, which exists in $(1, 4)$

So, it is differentiable in $(1, 4)$

Thus conditions of mean value theorem are satisfied.

Hence, there exists a real number $c \in (1, 4)$ such that

$$f'(c) = \frac{f(4) - f(1)}{4-1}$$

$$\Rightarrow \quad \frac{-4}{(4c-1)^2} = \frac{\frac{1}{16-1} - \frac{1}{4-1}}{4-1} = \frac{\frac{1}{15} - \frac{1}{3}}{3}$$

$$\Rightarrow \quad \frac{-4}{(4c-1)^2} = \frac{-4}{45}$$

$$\Rightarrow \quad (4c-1)^2 = 45$$

$$\Rightarrow \quad 4c - 1 = \pm 3\sqrt{5}$$

$$\Rightarrow \quad c = \frac{3\sqrt{5}+1}{4} \in (1, 4)$$

Hence, mean value theorem has been verified.