70. Discuss the applicability of Rolle's theorem on the function given by

$$f(x)=\begin{cases}x^2+1, & \text{if } 0\le x\le 1\\ 3-x, & \text{if } 1\le x\le 2\end{cases}$$

Sol. We have, $f(x)=\begin{cases}x^2+1, & \text{if } 0\le x\le 1\\ 3-x, & \text{if } 1\le x\le 2\end{cases}$

We know that, polynomial function is everywhere continuous and differentiability.

So, $f(x)$ is continuous and differentiable at all points except possibly at $x=1$.

Now $\lim\limits_{x\to 1^-}(x^2+1)=1+1=2$

And $\lim\limits_{x\to 1^+}(3-x)=3-1=2$

Also $f(1)=1^2+1=2$

So, $f(x)$ is continuous at $x=1$

Also $f'(x)=\begin{cases}2x, & \text{if } 0<x<1\\ -x, & \text{if } 1<x<2\end{cases}$

$$f'(1^-)=2(1)=2$$

and $$f'(1^+)=-1$$

Thus $f'(1^-)\neq f'(1^+)$.

So, $f(x)$ is not differentiable at $x=1$

Hence, Rolle's theorem is not applicable on the interval [0, 2]