

Example 17 Verify Rolle's theorem for the function, $f(x) = \sin 2x$ in $\left[0, \frac{\pi}{2}\right]$.

Solution Consider $f(x) = \sin 2x$ in $\left[0, \frac{\pi}{2}\right]$. Note that:

- (i) The function f is continuous in $\left[0, \frac{\pi}{2}\right]$, as f is a sine function, which is always continuous.
- (ii) $f'(x) = 2\cos 2x$, exists in $\left(0, \frac{\pi}{2}\right)$, hence f is derivable in $\left(0, \frac{\pi}{2}\right)$.
- (iii) $f(0) = \sin 0 = 0$ and $f\left(\frac{\pi}{2}\right) = \sin \pi = 0 \Rightarrow f(0) = f\left(\frac{\pi}{2}\right)$.

Conditions of Rolle's theorem are satisfied. Hence there exists at least one $c \in \left(0, \frac{\pi}{2}\right)$ such that $f'(c) = 0$. Thus

$$2 \cos 2c = 0 \quad \Rightarrow \quad 2c = \frac{\pi}{2} \quad \Rightarrow \quad c = \frac{\pi}{4}.$$