PROBLEM

If $y = (x + \sqrt{1+x^2})^n$, then $(1+x^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx}$ is [2002]

(a) n^2y (b) $-n^2y$ (c) $-y$ (d) $2x^2y$

(a) $y = (x+\sqrt{1+x^2})^n$

$$\frac{dy}{dx} = n(x+\sqrt{1+x^2})^{n-1}\left(1+\frac{1}{2}(1+x^2)^{-1/2}.\,2x\right);$$

$$\frac{dy}{dx} = n(x+\sqrt{1+x^2})^{n-1}\frac{(\sqrt{1+x^2}+x)}{\sqrt{1+x^2}}$$

:

or $\sqrt{1+x^2}\dfrac{dy}{dx} = ny$ or $\sqrt{1+x^2}\,y_1 = ny$

$\left(y_1 = \dfrac{dy}{dx}\right)$ Squaring, $(1+x^2)y_1^2 = n^2y^2$

Differentiating, $(1+x^2)2y_1y_2 + y_1^2.2x = n^2.2yy_1$

or $(1+x^2)y_2 + xy_1 = n^2y$