Assertion & Reason Type Questions

PROBLEM

Let f and g be real valued functions defined on interval $(-1, 1)$ such that $g''(x)$ is continuous, $g(0) \neq 0$. $g'(0) = 0$, $g''(0) \neq 0$, and $f(x) = g(x) \sin x$

STATEMENT - 1 : $\lim_{x \to 0} [g(x) \cot x - g(0) \operatorname{cosec} x] = f''(0)$

and

STATEMENT - 2 : $f'(0) = g(0)$ *(2008)*

(a) Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for Statement - 1

(b) Statement - 1 is True, Statement - 2 is True; Statement - 2 **is NOT** a correct explaination for Statement - 1

(c) Statement - 1 is True, Statement - 2 is False

(d) Statement - 1 is False, Statement - 2 is True

(a) We have $f(x) = g(x)\sin x$

$\Rightarrow \quad f'(x) = g'(x)\sin x + g(x)\cos x$

$\Rightarrow \quad f'(0) = g'(0) \times 0 + g(0) = g(0) \quad [\because g'(0) = 0]$

$\therefore$ Statement 2 is correct.

$$\text{Also } f''(0) = \lim_{x\to 0} \frac{f'(x) - f'(0)}{x}$$

$$= \lim_{x\to 0} \frac{g(x)\cos x + g'(x)\sin x - g(0)}{x}$$

$$= \lim_{x\to 0} \frac{g(x)\cos x - g(0)}{x} + \lim_{x\to 0} \frac{g'(x)\sin x}{x}$$

$$= \lim_{x\to 0} \frac{g(x)\cos x - g(0)}{x \times \frac{\sin x}{x}} + \lim_{x\to 0} g'(x)$$

$$= \lim_{x\to 0} \frac{g(x)\cos x - g(0)}{\sin x} + g'(0)$$

$$= \lim_{x\to 0} \left[g(x)\cot(x) - g(0)\operatorname{cosec} x\right] + 0$$

$$= \lim_{x\to 0} \left[g(x)\cot x - g(0)\operatorname{cosec} x\right]$$

$\therefore$ Statement 1 is also true and is a correct explanation for statement 2.