PROBLEM

If $f(x)$ is a twice differentiable function and given that $f(1)=1; f(2)=4, f(3)=9$, then *(2005S)*

(a) $f''(x)=2$ for $\forall\ x \in (1,3)$

(b) $f''(x)=f'(x)=5$ for some $x \in (2,3)$

(c) $f''(x)=3$ for $\forall\ x \in (2,3)$

(d) $f''(x)=2$ for some $x \in (1,3)$

(d) Let us consider the function $g(x)=f(x)-x^2$ so that

$$g(1)=f(1)-1^2=1-1=0$$
$$g(2)=f(2)-2^2=4-4=0$$
$$g(3)=f(3)-3^2=9-9=0$$

Since $f(x)$ is twice differentiable we can say $g(x)$ is continuous and differentiable everywhere and

$$g(1)=g(2)=g(3)=0$$

$\therefore$ By Rolle's theorem, $g'(c)=0$ for some $c \in (1,2)$

and $g'(d)=0$ for some $d \in (2,3)$

Again by Rolle's theorem,

$$g''(e)=0 \text{ for some } e \in (c,d) \Rightarrow e \in (1,3)$$

$\Rightarrow$ $f''(e)-2=0$ or $f''(e)=2$ for some $x \in (1,3)$

$f''(x)=2$ for some $x \in (1,3)$