

MCQs with One Correct Answer

1. If $y^2 = P(x)$, a polynomial of degree 3, then

$$2 \frac{d}{dx} \left(y^3 \frac{d^2 y}{dx^2} \right) \text{ equals} \quad (1988 - 2 \text{ Marks})$$

(a) $P'''(x) + P'(x)$

(b) $P''(x)P'''(x)$

(c) $P(x)P'''(x)$

(d) a constant

(c) We have $y^2 = P(x)$, ... (1)
where $P(x)$ is a polynomial of degree 3 and hence thrice differentiable. Differentiating (1) w.r. to x , we get

$$2y \frac{dy}{dx} = P'(x) \quad \dots (2)$$

Again differentiating with respect to x , we get

$$2 \left(\frac{dy}{dx} \right)^2 + 2y \frac{d^2y}{dx^2} = P''(x)$$

$$\Rightarrow \frac{[P'(x)]^2}{2y^2} + 2y \frac{d^2y}{dx^2} = P''(x) \quad [\text{Using (2)}]$$

$$\Rightarrow 4y^3 \frac{d^2y}{dx^2} = 2y^2 P''(x) - [P'(x)]^2$$

$$\Rightarrow 4y^3 \frac{d^2y}{dx^2} = 2P(x)P''(x) - [P'(x)]^2 \quad [\text{Using (1)}]$$

$$\Rightarrow 2y^3 \frac{d^2y}{dx^2} = P(x)P''(x) - \frac{1}{2}[P'(x)]^2$$

Again differentiating w.r. to x , we get $2 \frac{d}{dx} \left(y^3 \frac{d^2y}{dx^2} \right)$

$$= P'''(x)P(x) + P''(x)P'(x) - P'(x)P''(x) = P'''(x)P(x)$$