

81. If $x = \sin t$ and $y = \sin pt$, prove that $(1 - x^2) \frac{d^2 y}{dx^2} - \frac{dy}{dx} + p^2 y = 0$.

Sol. We have, $x = \sin t$ and $y = \sin pt$

$$\therefore \frac{dx}{dt} = \cos t \text{ and } \frac{dy}{dt} = p \cos pt$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{p \cdot \cos pt}{\cos t}$$

$$\Rightarrow y' = \frac{p \cdot \cos pt}{\cos t}$$

$$\Rightarrow y'^2 \cos^2 t = p^2 \cos^2 pt$$

$$\Rightarrow y'^2 (1 - \sin^2 t) = p^2 (1 - \sin^2 pt)$$

$$\Rightarrow y'^2 (1 - x^2) = p^2 (1 - y^2)$$

Differentiating above w.r.t. x , we get

$$2y'y''(1 - x^2) - 2xy'^2 = p^2(-2yy')$$

$$\Rightarrow y''(1 - x^2) - xy'^2 = -p^2 y$$

$$\Rightarrow y''(1 - x^2) - xy'^2 + p^2 y = 0$$