80. If $x^m . y^n = (x+y)^{m+n}$ then prove that (i) $\frac{dy}{dx} = \frac{y}{x}$ and (ii) $\frac{d^2y}{dx^2} = 0$

Sol. We have, $x^m \times y^n = (x+y)^{m+n}$

$\therefore \quad \log(x^m \times y^n) = \log[(x+y)^{m+n}]$

$\Rightarrow \quad m \log x + n \log y = (m+n) \log (x+y)$

Differentiating both sides w.r.t. x,

$$m\frac{1}{x} + n\frac{1}{y} \cdot \frac{dy}{dx} = (m+n)\frac{1}{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \quad \frac{m}{x} - \frac{m+n}{x+y} = \left(\frac{m+n}{x+y} - \frac{n}{y}\right)\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{my - nx}{x(x+y)} = \left(\frac{my - nx}{y(x+y)}\right)\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x}$$

Differentiating above w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{x\frac{dy}{dx} - y \cdot 1}{x^2}$$

$$= \frac{x \cdot \frac{y}{x} - y}{x^2} = 0$$