64. If $y = \tan^{-1} x$, find $\dfrac{d^2y}{dx^2}$ in terms of y alone.

Sol. We have, $y = \tan^{-1} x$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{1+x^2}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{d}{dx}(1+x^2)^{-1}$$

$$= -1(1+x^2)^{-2}\frac{d}{dx}(1+x^2)$$

$$= -\frac{1}{(1+x^2)^2} \cdot 2x$$

$$= \frac{-2\tan y}{(1+\tan^2 y)^2} \qquad [\because y = \tan^{-1} x \Rightarrow \tan y = x]$$

$$= \frac{-2\tan y}{(\sec^2 y)^2} = -2\frac{\sin y}{\cos y} \cdot \cos^2 y \cdot \cos^2 y = -\sin 2y \cdot \cos^2 y$$