Direction for Exercises 54 to 57:

Find dy/dx when x and y are connected by the relation given in each of these exercises.

54. $\sin(xy) + \dfrac{x}{y} = x^2 - y$

Sol. We have, $\sin(xy) + \dfrac{x}{y} = x^2 - y$

On differentiating both sides w.r.t x, we get

$$\cos xy \cdot \frac{d}{dx}(xy) + \frac{y\frac{d}{dx}(x) - x\frac{d}{dx}(y)}{y^2} = 2x - \frac{dy}{dx}$$

$$\Rightarrow \quad \cos xy\left[x\frac{dy}{dx} + y\right] + \frac{y - x\frac{dy}{dx}}{y^2} = 2x - \frac{dy}{dx}$$

$$\Rightarrow \quad \cos xy\left[xy^2\frac{dy}{dx} + y^3\right] + y - x\frac{dy}{dx} = 2xy^2 - \frac{dy}{dx}y^2$$

$$\Rightarrow \quad \frac{dy}{dx} = \left[\frac{2xy^2 - y^3\cos xy - y}{xy^2\cos xy - x + y^2}\right]$$