Example 14 If $y = \tan x + \sec x$, prove that $\dfrac{d^2 y}{dx^2} = \dfrac{\cos x}{(1 - \sin x)^2}$.

Solution We have $y = \tan x + \sec x$. Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \sec^2 x + \sec x \tan x$$

$$= \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x} = \frac{1 + \sin x}{\cos^2 x} = \frac{1 + \sin x}{(1 + \sin x)(1 - \sin x)}.$$

thus $\dfrac{dy}{dx} = \dfrac{1}{1 - \sin x}$.

Now, differentiating again w.r.t. x, we get

$$\frac{d^2 y}{dx^2} = \frac{-(-\cos x)}{(1 - \sin x)^2} = \frac{\cos x}{(1 - \sin x)^2}$$