PROBLEM

If $A > 0, B > 0$, and $A + B = \frac{\pi}{3}$, then the maximum value of $\tan A \tan B$ is

a. $\frac{1}{\sqrt{3}}$ **b.** $\frac{1}{3}$

c. 3 **d.** $\sqrt{3}$

SOLUTION

b. Given $A + B = 60°$ or $B = 60° - A$

$$\therefore \tan B = \tan (60° - A) = \frac{\sqrt{3} - \tan A}{1 + \sqrt{3} \tan A}$$

Now $z = \tan A \tan B$

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$$\text{or } z = \frac{t(\sqrt{3} - t)}{1 + \sqrt{3}\, t} = \frac{\sqrt{3}\, t - t^2}{1 + \sqrt{3}\, t}$$

where $t = \tan A$

$$\frac{dz}{dt} = -\frac{(t + \sqrt{3})(\sqrt{3}t - 1)}{(1 + \sqrt{3}\, t)^2} = 0$$

or $t = 1/\sqrt{3}$

or $t = \tan A = \tan 30°$

The other value is rejected as both A and B are +ve acute angles.

If $t < \frac{1}{\sqrt{3}}$, $\frac{dz}{dt}$ is positive and if $t > \frac{1}{\sqrt{3}}$, $\frac{dz}{dt}$ is negative.

Hence, maximum when $t = \frac{1}{\sqrt{3}}$ and maximum value $= \frac{1}{3}$.