PROBLEM

The greatest value of $f(x) = (x + 1)^{1/3} - (x - 1)^{1/3}$ on $[0, 1]$ is

a. 1 **b.** 2

c. 3 **d.** $\frac{1}{3}$

SOLUTION

b. We have $f(x) = (x + 1)^{1/3} - (x - 1)^{1/3}$

$$\therefore f'(x) = \frac{1}{3}(x+1)^{\frac{-2}{3}} - \frac{1}{3}(x-1)^{\frac{-2}{3}}$$

$$= \frac{(x-1)^{2/3} - (x+1)^{2/3}}{3(x^2-1)^{2/3}}$$

Clearly, $f'(x)$ does not exist at $x = \pm 1$. Now,

$f'(x) = 0$

or $(x - 1)^{2/3} = (x + 1)^{2/3}$

or $(x - 1)^2 = (x + 1)^2$

or $-2x = 2x$ or $4x = 0$ or $x = 0$

Clearly, $f'(x) \neq 0$ for any other values of $x \in [0, 1]$.

The value of $f(x)$ at $x = 0$ is 2.

Hence, the greatest value of $f(x)$ is 2.