

PROBLEM

If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$, attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals [2003]

- (a) $\frac{1}{2}$ (b) 3 (c) 1 (d) 2

SOLUTION

(d) $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$

$$f'(x) = 6x^2 - 18ax + 12a^2; f''(x) = 12x - 18a$$

For max. or min.

$$6x^2 - 18ax + 12a^2 = 0 \Rightarrow x^2 - 3ax + 2a^2 = 0$$

$\Rightarrow x = a$ or $x = 2a$. At $x = a$ max. and at $x = 2a$ min

$$\therefore p = a \text{ and } q = 2a$$

As per question $p^2 = q$

$$\therefore a^2 = 2a \Rightarrow a = 2 \text{ or } a = 0$$

but $a > 0$, therefore, $a = 2$.