

Integer Value Correct Type

PROBLEM

The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set $A = \{x \mid x^2 + 20 \leq 9x\}$ is (2009)

SOLUTION

- (7) The given function is $f(x) = 2x^3 - 15x^2 + 36x - 48$
and $A = \{x \mid x^2 + 20 \leq 9x\}$
 $\Rightarrow A = \{x \mid x^2 - 9x + 20 \leq 0\}$
 $\Rightarrow A = \{x \mid (x-4)(x-5) \leq 0\}$
 $\Rightarrow A = [4, 5]$

Also $f'(x) = 6x^2 - 30x + 36 = 6(x^2 - 5x + 6)$
 $= 6(x-2)(x-3)$

Clearly $\forall x \in A, f'(x) > 0$

$\therefore f$ is strictly increasing function on A .

\therefore Maximum value of f on A

$$= f(5) = 2 \times 5^3 - 15 \times 5^2 + 36 \times 5 - 48$$
$$= 250 - 375 + 180 - 48 = 430 - 423 = 7$$