PROBLEM

The number of values of x where the function $f(x) = \cos x + \cos(\sqrt{2}x)$ attains its maximum is

(1998 - 2 Marks)

(a) 0 (b) 1

(c) 2 (d) infinite

SOLUTION

To show

$$1 + x\ln(x + \sqrt{x^2+1}) \geq \sqrt{1+x^2} \text{ for } x \geq 0$$

Consider $f(x) = 1 + x\ln(x + \sqrt{x^2+1}) - \sqrt{1+x^2})$

Here, $f'(x) = \ln(x + \sqrt{x^2+1}) + \dfrac{x}{x + \sqrt{x^2+1}}$

$$\left[1 + \frac{x}{\sqrt{x^2+1}}\right] - \frac{x}{\sqrt{1+x^2}}$$

$= \ln(x + \sqrt{x^2+1})$

As $x + \sqrt{x^2+1} \geq 1$ for $x \geq 1$

$\therefore \quad \ln(x + \sqrt{x^2+1}) \geq 0$

$\therefore \quad f'(x) \geq 0, \forall\, x \geq 0$

Hence $f(x)$ is increasing function.

Now for $x \geq 0 \Rightarrow f(x) \geq f(0)$

$\Rightarrow \quad 1 + x\ln(x + \sqrt{x^2+1}) - \sqrt{1+x^2} \geq 0$

$\Rightarrow \quad 1 + x\ln(x + \sqrt{x^2+1}) \geq \sqrt{1+x^2}$