D MCQs with One or More than One Correct

PROBLEM

Let $P(x) = a_0 + a_1x^2 + a_2x^4 + ... + a_nx^{2n}$ be a polynomial in a real variable x with

$0 < a_0 < a_1 < a_2 < ..... < a_n$. The function $P(x)$ has

(a) neither a maximum nor a minimum *(1986 - 2 Marks)*
(b) only one maximum
(c) only one minimum
(d) only one maximum and only one minimum
(e) none of these.

SOLUTION

$$f(x)=\frac{(a+x)(b+x)}{(c+x)}, a,b>c, x>-c$$

$$=\frac{(a-c+x+c)(b-c+x+c)}{x+c}$$

$$=\frac{(a-c)(b-c)}{x+c}+(x+c)+a+b-2c$$

$$\Rightarrow \quad f'(x)=\frac{-(a-c)(b-c)}{(x+c)^2}+1$$

$$\therefore \quad f'(x)=0 \Rightarrow x=-c\pm\sqrt{(a-c)(b-c)}$$

$\Rightarrow \quad x=-c+\sqrt{(a-c)(b-c)}$ [+ve sign is taken $\because x>-c$]

Also $f''(x)=\dfrac{2(a-c)(b-c)}{(x+c)^3}>0$ for a, $b>c$ and $x>-c$

$\therefore \quad f(x)$ is least at $x=-c+\sqrt{(a-c)(b-c)}$

$$\therefore \quad f_{\min}=\frac{(a-c)(b-c)}{\sqrt{(a-c)(b-c)}}+\sqrt{(a-c)(b-c)}$$

$$+(a-c)+(b-c)$$

$$=(a-c)+(b-c)+2\sqrt{(a-c)(b-c)}$$

$$=(\sqrt{a-c}+\sqrt{b-c})^2$$