PROBLEM

Let $f(x) = (1 + b^2)x^2 + 2bx + 1$ and let m(b) be the minimum value of $f(x)$. As b varies, the range of m(b) is *(2001S)*

(a) [0, 1] (b) (0, 1/2] (c) [1/2, 1] (d) (0, 1]

SOLUTION

(d) $f(x) = (1 + b^2)x^2 + 2bx + 1$

It is a quadratic expression with coeff. of $x^2 = 1 + b^2 > 0$.

$\therefore$ $f(x)$ represents an upward parabola whose min value is $\frac{-D}{4a}$, D being the discreminant.

$$\therefore \quad m(b) = -\frac{4b^2 - 4(1+b^2)}{4(1+b^2)} \Rightarrow m(b) = \frac{1}{1+b^2}$$

For range of m (b) :

$\frac{1}{1+b^2} > 0$ also $b^2 \geq 0 \Rightarrow 1 + b^2 \geq 1$

$\Rightarrow \frac{1}{1+b^2} \leq 1$. Thus $m(b) = (0, 1]$