PROBLEM

Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$. Then which one of the following holds? **[2008]**

(a) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$

(b) The cubic has minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$

(c) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

(d) The cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

SOLUTION

(a) Let $y = x^3 - px + q \Rightarrow \frac{dy}{dx} = 3x^2 - p$

For $\frac{dy}{dx} = 0 \Rightarrow 3x^2 - p = 0 \Rightarrow x = \pm\sqrt{\frac{p}{3}}$

$\frac{d^2y}{dx^2} = 6x$

$$\left.\frac{d^2y}{dx^2}\right|_{x=\sqrt{\frac{p}{3}}} = +ve \text{ and } \left.\frac{d^2y}{dx^2}\right|_{x=-\sqrt{\frac{p}{3}}} = -ve$$

$\therefore$ y has minima at $x = \sqrt{\frac{p}{3}}$ and maxima at $x = -\sqrt{\frac{p}{3}}$