

PROBLEM

If  $p$  and  $q$  are positive real numbers such that  $p^2 + q^2 = 1$ , then the maximum value of  $(p + q)$  is [2007]

- (a)  $\frac{1}{2}$       (b)  $\frac{1}{\sqrt{2}}$       (c)  $\sqrt{2}$       (d) 2.

SOLUTION

(c) Given that  $p^2 + q^2 = 1 \quad \therefore p = \cos \theta$  and  $q = \sin \theta$

Then  $p + q = \cos \theta + \sin \theta$

We know that

$$-\sqrt{a^2 + b^2} \leq a \cos \theta + b \sin \theta \leq \sqrt{a^2 + b^2}$$

$$\therefore -\sqrt{2} \leq \cos \theta + \sin \theta \leq \sqrt{2}$$

Hence max. value of  $p + q$  is  $\sqrt{2}$