JEE ADVANCED/IIT-JEE

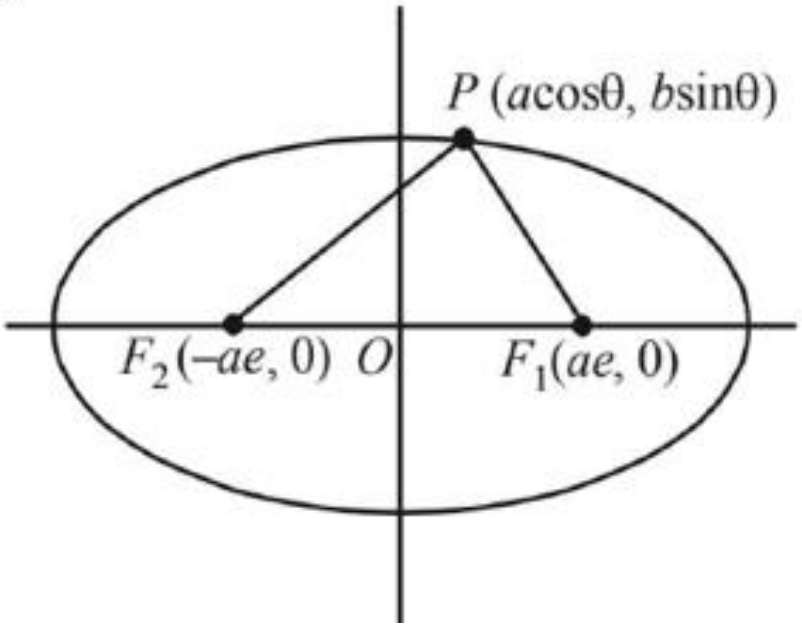
A Fill in the Blanks

PROBLEM

Let P be a variable point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with foci F_1 and F_2. If A is the area of the triangle PF_1F_2 then the maximum value of A is ..... ***(1994 - 2 Marks)***

SOLUTION

Let $P(a\cos\theta, b\sin\theta)$ be any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with foci $F_1(ae, 0)$ and $F_2(-ae, 0)$



Then area of ΔPF_1F_2 is given by

$$A = \frac{1}{2}\begin{vmatrix} a\cos\theta & b\sin\theta & 1 \\ ae & 0 & 1 \\ -ae & 0 & 1 \end{vmatrix}$$

$$= \frac{1}{2}|-b\sin\theta(ae+ae)| = abe\,|\sin\theta|$$

$\because \quad |\sin\theta| \le 1$

$\therefore \quad A_{\max} = abe$