64. The least value of the function $f(x) = ax + \dfrac{b}{x}$ $(a > 0, b > 0, x > 0)$ is _____.

Sol. $f(x) = ax + \dfrac{b}{x}$

$$\Rightarrow \quad f'(x) = a - \dfrac{b}{x^2}$$

$f'(x) = 0$

$$\Rightarrow \quad a = \dfrac{b}{x^2} \text{ or } x = \sqrt{\dfrac{b}{a}} \quad (\text{as } x > 0)$$

Now, $f''(x) = \dfrac{2b}{x^3} > 0$ for $x = \sqrt{\dfrac{b}{a}}$

Thus least value of $f(x)$ is $f\left(\sqrt{\dfrac{b}{a}}\right) = a \cdot \sqrt{\dfrac{b}{a}} + \dfrac{b}{\sqrt{\dfrac{b}{a}}} = \sqrt{ab} + \sqrt{ab} = 2\sqrt{ab}$