

32. AB is a diameter of a circle and C is any point on the circle. Show that the area of $\triangle ABC$ is maximum, when it is isosceles.

Sol. We have, $AB = 2r$

And $\angle ACB = 90^\circ$

Let $\angle ABC = \theta$

$\Rightarrow AC = 2r \sin \theta$ and $BC = 2r \cos \theta$

$$\begin{aligned}\therefore \text{Area of } \triangle ABC, \Delta &= \frac{1}{2} (2r \sin \theta)(2r \cos \theta) \\ &= r^2 \sin 2\theta\end{aligned}$$

Clearly, Δ is maximum when $\sin 2\theta$ is maximum.

$$\therefore \Delta_{\max} = r^2, \text{ when } \sin 2\theta = 1$$

$$\text{or } 2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$$

\Rightarrow Triangle is isosceles.

