

30. Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible, when revolved about one of its sides. Also find the maximum volume.

Sol. Let breadth and length of the rectangle be x and y , respectively.

It is given that the perimeter of the rectangle = 36 cm

$$\Rightarrow 2x + 2y = 36$$

$$\Rightarrow x + y = 18$$

$$\Rightarrow y = 18 - x$$

Let the rectangle is being revolved about its length y .

Then, volume of resultant cylinder,

$$\Rightarrow V = \pi x^2 y = \pi x^2 (18 - x) = \pi (18x^2 - x^3)$$

$$\Rightarrow \frac{dV}{dx} = \pi (36x - 3x^2)$$

$$\frac{dV}{dx} = 0$$

$$\Rightarrow 36x = 3x^2$$

$$\Rightarrow x = 12 \quad (\text{as } x \neq 0)$$

$$\text{Also } \frac{d^2V}{dx^2} = \pi (36 - 6x)$$

$$\Rightarrow \left(\frac{d^2V}{dx^2} \right)_{x=12} = \pi (36 - 72) = -36\pi < 0$$

\therefore For $x = 12$, volume of the resultant cylinder is the maximum.

$$\therefore y = 18 - 12 = 6$$

So, the dimensions of rectangle are 12 cm and 6 cm, respectively.

\therefore Maximum volume of resultant cylinder,

$$\begin{aligned} V &= \pi [18 \cdot (12)^2 - (12)^3] \\ &= \pi 12^2 (18 - 12) \\ &= 864\pi \text{ cm}^3 \end{aligned}$$

