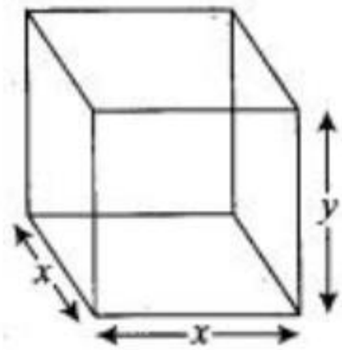
29. An open box with square base is to be made of a given quantity of card board of area c^2. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

Sol. Let the length of side of the square base of open box be x unit and its height be y units.



$\therefore$ Area of the card board used $= x^2 + 4xy$

$\therefore$ $x^2 + 4xy = c^2$ (given)

$$\Rightarrow \quad y = \frac{c^2 - x^2}{4x} \qquad \text{(i)}$$

Now, volume of the box, $V = x^2 y$

$$\Rightarrow \quad V = x^2\left(\frac{c^2 - x^2}{4x}\right) = \frac{1}{4}x(c^2 - x^2) = \frac{1}{4}(c^2 x - x^3)$$

$$\Rightarrow \quad \frac{dV}{dx} = \frac{1}{4}(c^2 - 3x^2) \qquad \text{(ii)}$$

$$\Rightarrow \quad \frac{dV}{dx} = 0 \Rightarrow c^2 = 3x^2 \quad \Rightarrow \quad x = \frac{c}{\sqrt{3}}$$

Differentiating (ii) w.r.t. x, we get

$$\frac{d^2V}{dx^2} = \frac{1}{4}(-6x) = -\frac{3}{2}x < 0$$

$\therefore$ $x = \frac{c}{\sqrt{3}}$ is point of maxima

$$\therefore \text{ Maximum volume } = \frac{1}{4}\left(c^2 \cdot \frac{c}{\sqrt{3}} - \frac{c^3}{3\sqrt{3}}\right) = \frac{1}{4} \cdot \frac{(3c^3 - c^3)}{3\sqrt{3}} = \frac{c^3}{6\sqrt{3}} \text{ cu units}$$