Example 25 If $f(x) = \dfrac{1}{4x^2 + 2x + 1}$, then its maximum value is _____.

Solution For f to be maximum, $4x^2 + 2x + 1$ should be minimum i.e.

$$4x^2 + 2x + 1 = 4\left(x + \frac{1}{4}\right)^2 + \left(1 - \frac{1}{4}\right)$$ giving the minimum value of $4x^2 + 2x + 1 = \dfrac{3}{4}$.

Hence maximum value of $f = \dfrac{4}{3}$.