Example 17 Find the difference between the greatest and least values of the function $f(x) = \sin 2x - x$, on $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$.

Solution $f(x) = \sin 2x - x$

$\Rightarrow \qquad f'(x) = 2\cos 2x - 1$

Therefore, $f'(x) = 0 \Rightarrow \cos 2x = \dfrac{1}{2} \Rightarrow 2x$ is $\dfrac{-\pi}{3}$ or $\dfrac{\pi}{3} \Rightarrow x = -\dfrac{\pi}{6}$ or $\dfrac{\pi}{6}$

$$f\left(-\dfrac{\pi}{2}\right) = \sin(-\pi) + \dfrac{\pi}{2} = \dfrac{\pi}{2}$$

$$f\left(-\dfrac{\pi}{6}\right) = \sin\left(-\dfrac{2\pi}{6}\right) + \dfrac{\pi}{6} = -\dfrac{\sqrt{3}}{2} + \dfrac{\pi}{6}$$

$$f\left(\dfrac{\pi}{6}\right) = \sin\left(\dfrac{2\pi}{6}\right) - \dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2} - \dfrac{\pi}{6}$$

$$f\left(\dfrac{\pi}{2}\right) = \sin(\pi) - \dfrac{\pi}{2} = -\dfrac{\pi}{2}$$

Clearly, $\dfrac{\pi}{2}$ is the greatest value and $-\dfrac{\pi}{2}$ is the least.

Therefore, difference $= \dfrac{\pi}{2} + \dfrac{\pi}{2} = \pi$