

## MULTIPLE CORRECT ANSWER

The value of  $k$  ( $k > 0$ ) such that the length of the longest interval in which the function  $f(x) = \sin^{-1} |\sin kx| + \cos^{-1}(\cos kx)$  is constant is  $\pi/4$  is/are

- (1) 8                      (2) 4                      (3) 12                      (4) 16

### SOLUTION

(2)  $f(x) = \sin^{-1} |\sin kx| + \cos^{-1}(\cos kx)$

Let  $g(x) = \sin^{-1} |\sin x| + \cos^{-1}(\cos x)$

$$g(x) = \begin{cases} 2x, & 0 \leq x \leq \frac{\pi}{2} \\ \pi, & \frac{\pi}{2} < x \leq \frac{3\pi}{2} \\ 4\pi - 2x, & \frac{3\pi}{2} < x \leq 2\pi \end{cases}$$

$g(x)$  is periodic with period  $2\pi$  and is constant in the continuous interval  $\left[2n\pi + \frac{\pi}{2}, 2n\pi + \frac{3\pi}{2}\right]$  (where  $n \in I$ ) and  $f(x) = g(kx)$ .

So,  $f(x)$  is constant in the interval

$$\left[\frac{2n\pi}{k} + \frac{\pi}{2k}, \frac{2n\pi}{k} + \frac{3\pi}{2k}\right]$$

Thus,  $\frac{\pi}{4} = \frac{3\pi}{2k} - \frac{\pi}{2k}$

or  $\frac{\pi}{k} = \frac{\pi}{4}$

or  $k = 4$