MULTIPLE CORRECT ANSWER

If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, then

(1) $x^2 + y^2 + z^2 + 2xyz = 1$

(2) $2(\sin^{-1}x + \sin^{-1}y + \sin^{-1}z) = \cos^{-1}x + \cos^{-1}y + \cos^{-1}z$

(3) $xy + yz + zx = x + y + z - 1$

(4) $\left(x + \frac{1}{x}\right) + \left(y + \frac{1}{y}\right) + \left(z + \frac{1}{z}\right) \geq 6$

SOLUTION

(1), (2)

$$\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$$

or $\quad \sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi/2$

or $\quad \cos^{-1}x + \cos^{-1}y = \cos^{-1}(-z)$

$\Rightarrow \quad xy - \sqrt{1-x^2}\sqrt{1-y^2} = -z$

or $\quad x^2 + y^2 + z^2 + 2xyz = 1$