If $\sin^{-1}\left(\dfrac{4x}{x^2+4}\right) + 2\tan^{-1}\left(-\dfrac{x}{2}\right)$ is independent of x, find the values of x.

SOLUTION

$$E = \sin^{-1}\left(\dfrac{4x}{x^2+4}\right) + 2\tan^{-1}\left(-\dfrac{x}{2}\right)$$

$$= \sin^{-1}\left(\dfrac{2\times\dfrac{x}{2}}{\left(\dfrac{x}{2}\right)^2+1}\right) - 2\tan^{-1}\dfrac{x}{2}$$

$$= 2\tan^{-1}\dfrac{x}{2} - 2\tan^{-1}\dfrac{x}{2}$$

$$= 0 \qquad\qquad \text{(For } E \text{ to be independent of } x)$$

$$\Rightarrow \qquad \left|\dfrac{x}{2}\right| \le 1$$

or $\qquad |x| \le 2$ or $-2 \le x \le 2$