

## SINGLE CORRECT ANSWER

The value of

$$\frac{\alpha^3}{2} \operatorname{cosec}^2 \left( \frac{1}{2} \tan^{-1} \left( \frac{\alpha}{\beta} \right) \right) + \frac{\beta^3}{2} \sec^2 \left( \frac{1}{2} \tan^{-1} \left( \frac{\beta}{\alpha} \right) \right)$$

is equal to

(1)  $(\alpha - \beta)(\alpha^2 + \beta^2)$

(2)  $(\alpha + \beta)(\alpha^2 - \beta^2)$

(3)  $(\alpha + \beta)(\alpha^2 + \beta^2)$

(4) none of these

## SOLUTION

$$4. (3) \quad \frac{\alpha^3}{2} \operatorname{cosec}^2 \left( \frac{1}{2} \tan^{-1} \frac{\alpha}{\beta} \right) + \frac{\beta^3}{2} \sec^2 \left( \frac{1}{2} \tan^{-1} \frac{\beta}{\alpha} \right)$$

$$= \alpha^3 \frac{1}{1 - \cos \left( \tan^{-1} \left( \frac{\alpha}{\beta} \right) \right)} + \beta^3 \frac{1}{1 + \cos \left( \tan^{-1} \frac{\beta}{\alpha} \right)}$$

$$= \alpha^3 \frac{1}{1 - \cos \left( \cos^{-1} \left( \frac{\beta}{\sqrt{\alpha^2 + \beta^2}} \right) \right)} + \beta^3 \frac{1}{1 + \cos \left( \cos^{-1} \frac{\alpha}{\sqrt{\alpha^2 + \beta^2}} \right)}$$

$$= \alpha^3 \frac{1}{1 - \frac{\beta}{\sqrt{\alpha^2 + \beta^2}}} + \beta^3 \frac{1}{1 + \frac{\alpha}{\sqrt{\alpha^2 + \beta^2}}}$$

$$= \sqrt{\alpha^2 + \beta^2} \left( \frac{\alpha^3}{\sqrt{\alpha^2 + \beta^2} - \beta} + \frac{\beta^3}{\sqrt{\alpha^2 + \beta^2} + \alpha} \right)$$

$$= \sqrt{\alpha^2 + \beta^2} \left( \alpha^3 \frac{(\sqrt{\alpha^2 + \beta^2} + \beta)}{\alpha^2} + \beta^3 \frac{(\sqrt{\alpha^2 + \beta^2} - \alpha)}{\beta^2} \right)$$

$$= \sqrt{\alpha^2 + \beta^2} \left[ \alpha (\sqrt{\alpha^2 + \beta^2} + \beta) + \beta (\sqrt{\alpha^2 + \beta^2} - \alpha) \right]$$

$$= \sqrt{\alpha^2 + \beta^2} (\alpha + \beta) \sqrt{\alpha^2 + \beta^2}$$

$$= (\alpha + \beta)(\alpha^2 + \beta^2)$$