SINGLE CORRECT ANSWER

If $A = 2 \tan^{-1}(2\sqrt{2} - 1)$ and $B = 3 \sin^{-1}\left(\dfrac{1}{3}\right) + \sin^{-1}\left(\dfrac{3}{5}\right)$, then which is greater ?

Sol. We have $A = 2 \tan^{-1}(2\sqrt{2} - 1) = 2 \tan^{-1}(1.828)$

$\Rightarrow \qquad A > 2 \tan^{-1}\sqrt{3}$

$\Rightarrow \qquad A > \dfrac{2\pi}{3}$

Now, $\sin^{-1}\left(\dfrac{1}{3}\right) < \sin^{-1}\left(\dfrac{1}{2}\right)$

$\Rightarrow \qquad \sin^{-1}\left(\dfrac{1}{3}\right) < \dfrac{\pi}{6}$

$\Rightarrow \qquad 3\sin^{-1}\dfrac{1}{3} < \dfrac{\pi}{2}$

Further, $\sin^{-1}\left(\dfrac{3}{5}\right) = \sin^{-1}(0.6) < \sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

$\Rightarrow \qquad \sin^{-1}\left(\dfrac{3}{5}\right) < \dfrac{\pi}{3}$

$\Rightarrow \qquad B = 3\sin^{-1}\left(\dfrac{1}{3}\right) + \sin^{-1}\left(\dfrac{3}{5}\right) < \dfrac{\pi}{2} + \dfrac{\pi}{3}$

$\Rightarrow \qquad B < \dfrac{5\pi}{6}$

From this, we really cannot relate A and B.

Now, $3 \sin^{-1}\left(\dfrac{1}{3}\right) = \sin^{-1}\left[3.\dfrac{1}{3} - 4\left(\dfrac{1}{3}\right)^3\right]$

$$= \sin^{-1}\left(\dfrac{23}{27}\right)$$

$$= \sin^{-1}(0.852)$$

$\Rightarrow \qquad 3 \sin^{-1}\left(\dfrac{1}{3}\right) < \sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = \dfrac{\pi}{3}$

Hence, $B = 3 \sin^{-1}\left(\dfrac{1}{3}\right) + \sin^{-1}\left(\dfrac{3}{5}\right) < \dfrac{\pi}{3} + \dfrac{\pi}{3} = \dfrac{2\pi}{3}$

$\therefore \qquad A > B.$