SINGLE CORRECT ANSWER

Solve $\cos^{-1}\left(\frac{1}{2}x^2+\sqrt{1-x^2}\sqrt{1-\frac{x^2}{4}}\right)=\cos^{-1}\frac{x}{2}-\cos^{-1}x$.

SOLUTION

Sol. $\cos^{-1}\left(\frac{1}{2}x^2+\sqrt{1-x^2}\sqrt{1-\frac{x^2}{4}}\right)$

$$=\cos^{-1}\left(x\cdot\frac{x}{2}+\sqrt{1-x^2}\sqrt{1-\left(\frac{x}{2}\right)^2}\right)$$

For $\cos^{-1}\left(\frac{1}{2}x^2+\sqrt{1-x^2}\sqrt{1-\frac{x^2}{4}}\right)=\cos^{-1}\frac{x}{2}-\cos^{-1}x$,

L.H.S. > 0, hence R.H.S. > 0

$\Rightarrow\quad \cos^{-1}\frac{x}{2}-\cos^{-1}x>0$ or $\cos^{-1}\frac{x}{2}>\cos^{-1}x$

Since $\cos^{-1}x$ is a decreasing function, we get

$$\frac{x}{2}\le x \;\Rightarrow\; \frac{x}{2}\ge 0 \;\Rightarrow\; x\ge 0 \;\Rightarrow\; x\in[0,1]$$