If $x \in \left(0, \dfrac{\pi}{2}\right)$, then show that

$$\cos^{-1}\left(\dfrac{7}{2}(1 + \cos 2x) + \sqrt{(\sin^2 x - 48\cos^2 x)\sin x}\right)$$
$$= x - \cos^{-1}(7\cos x)$$

SOLUTION

$$y = \cos^{-1}\left(\dfrac{7}{2}(1 + \cos 2x) + \sqrt{(\sin^2 x - 48\cos^2 x)\sin x}\right)$$

$$= \cos^{-1}\left((7\cos x)(\cos x) + \sqrt{1 - 49\cos^2 x}\sqrt{1 - \cos^2 x}\right)$$

$$= \cos^{-1}(\cos x) - \cos^{-1}(7\cos x) \qquad \left[\because \cos x < 7\cos x \text{ for } x \in \left(0, \dfrac{\pi}{2}\right)\right]$$

$$= x - \cos^{-1}(7\cos x)$$