PROBLEM

If $\cos^{-1} x - \cos^{-1} \frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to **[2005]**

(a) $2\sin 2\alpha$ (b) 4

(c) $4\sin^2\alpha$ (d) $-4\sin^2\alpha$

SOLUTION

(c) $\cos^{-1} x - \cos^{-1}\frac{y}{2} = \alpha$

$$\cos^{-1}\left\{\frac{xy}{2} + \sqrt{(1-x^2)\left(1-\frac{y^2}{4}\right)}\right\} = \alpha$$

$$\cos^{-1}\left\{\frac{xy + \sqrt{4 - y^2 - 4x^2 + x^2y^2}}{2}\right\} = \alpha$$

$$\Rightarrow \quad 4 - y^2 - 4x^2 + x^2y^2$$

$$= 4\cos^2\alpha + x^2y^2 - 4xy\cos\alpha$$

$$\Rightarrow \quad 4x^2 + y^2 - 4xy\cos\alpha = 4\sin^2\alpha.$$