

PROBLEM

$$\cot^{-1}(\sqrt{\cos \alpha}) - \tan^{-1}(\sqrt{\cos \alpha}) = x, \text{ then } \sin x =$$

(a) $\tan^2\left(\frac{\alpha}{2}\right)$

(b) $\cot^2\left(\frac{\alpha}{2}\right)$

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(c) $\tan \alpha$

(d) $\cot\left(\frac{\alpha}{2}\right)$

(a) $\cot^{-1}(\sqrt{\cos \alpha}) - \tan^{-1}(\sqrt{\cos \alpha}) = x$

$$\tan^{-1}\left(\frac{1}{\sqrt{\cos \alpha}}\right) - \tan^{-1}(\sqrt{\cos \alpha}) = x$$

$$\Rightarrow \tan^{-1} \frac{\frac{1}{\sqrt{\cos \alpha}} - \sqrt{\cos \alpha}}{1 + \frac{1}{\sqrt{\cos \alpha}} \cdot \sqrt{\cos \alpha}} = x$$

$$\Rightarrow \tan^{-1} \frac{1 - \cos \alpha}{2\sqrt{\cos \alpha}} = x$$

$$\Rightarrow \tan x = \frac{1 - \cos \alpha}{2\sqrt{\cos \alpha}} \quad \text{or} \quad \cot x = \frac{2\sqrt{\cos \alpha}}{1 - \cos \alpha}$$

$$\Rightarrow \sin x = \frac{1 - \cos \alpha}{1 + \cos \alpha} = \frac{1 - (1 - 2 \sin^2 \alpha / 2)}{1 + 2 \cos^2 \alpha / 2 - 1}$$

$$\text{or } \sin x = \tan^2 \frac{\alpha}{2}$$