Match the Following

PROBLEM

DIRECTIONS (Q. 3) : *Following question has matching lists. The codes for the lists have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

Match List I with List II and select the correct answer using the code given below the lists :

List I	List II
P. $\left(\frac{1}{y^2}\left(\frac{\cos(\tan^{-1}y)+y\sin(\tan^{-1}y)}{\cot(\sin^{-1}y)+\tan(\sin^{-1}y)}\right)^2+y^4\right)^{1/2}$ takes value	1. $\frac{1}{2}\sqrt{\frac{5}{3}}$
Q. If $\cos x+\cos y+\cos z=0=\sin x+\sin y+\sin z$ then possible value of $\cos\frac{x-y}{2}$ is	2. $\sqrt{2}$
R. If $\cos\left(\frac{\pi}{4}-x\right)\cos 2x+\sin x\sin 2\sec x=\cos x\sin 2x\sec x+\cos\left(\frac{\pi}{4}+x\right)\cos 2x$ then possible value of $\sec x$ is	3. $\frac{1}{2}$
S. If $\cot\left(\sin^{-1}\sqrt{1-x^2}\right)=\sin\left(\tan^{-1}\left(x\sqrt{6}\right)\right), x\neq 0$, then possible value of x is	4. 1

Codes:

	P	**Q**	**R**	**S**
(a)	4	3	1	2
(b)	4	3	2	1
(c)	3	4	2	1
(d)	3	4	1	2

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SOLUTION

(b)

(P) $$\left[\frac{1}{y^2}\left(\frac{\cos(\tan^{-1}y)+y\sin(\tan^{-1}y)}{\cot(\sin^{-1}y)+\tan(\sin^{-1}y)}\right)^2+y^4\right]^{\frac{1}{2}}$$

$$=\left[\frac{1}{y^2}\left(\frac{\cos\left(\cos^{-1}\frac{1}{\sqrt{1+y^2}}\right)+y\sin\left(\sin^{-1}\frac{y}{\sqrt{1+y^2}}\right)}{\cot\left(\cot^{-1}\frac{\sqrt{1-y^2}}{y}\right)+\tan\left(\tan^{-1}\frac{y}{\sqrt{1-y^2}}\right)}\right)^2+y^4\right]^{\frac{1}{2}}$$

$$=\left[\frac{1}{y^2}\left(\frac{\sqrt{1+y^2}}{\frac{1}{y(\sqrt{1-y^2})}}\right)^2+y^4\right]^{\frac{1}{2}}$$

$$=\left(1-y^4+y^4\right)^{\frac{1}{2}}=1 \qquad \therefore \quad (P)\rightarrow(4)$$

(Q) We have $\cos x+\cos y=-\cos z$

$\sin x+\sin y=-\sin z$

Squaring and adding we get

$(\cos x + \cos y)^2 + (\sin x + \sin y)^2 = \cos^2 z + \sin^2 z$

$\Rightarrow \quad 2 + 2\cos(x - y) = 1$

$\Rightarrow \quad 4\cos^2 \frac{x-y}{2} = 1 \quad \text{or} \quad \cos\frac{x-y}{2} = \frac{+1}{2}$

$\therefore \quad Q \to (3)$

(R) We have

$\cos\left(\frac{\pi}{4} - x\right)\cos 2x + \sin x \sin 2x \sec x$

$= \cos x \sin 2x \sec x + \cos\left(\frac{\pi}{4} + x\right)\cos 2x$

$\Rightarrow \cos 2x\left[\cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{\pi}{4} + x\right)\right]$

$= \sin 2x \sec x (\cos x - \sin x)$

$\Rightarrow 2\sin\frac{\pi}{4}\sin x \cos 2x = 2\sin x(\cos x - \sin x)$

$\Rightarrow 2\sin x\left[\frac{1}{\sqrt{2}}(\cos^2 x - \sin^2 x) - (\cos x - \sin x)\right] = 0$

$\Rightarrow 2\sin x(\cos x - \sin x)\left(\frac{\cos x + \sin x}{\sqrt{2}} - 1\right) = 0$

$\Rightarrow \sin x = 0 \text{ or } \tan x = 1 \text{ or } \cos\left(x - \frac{\pi}{4}\right) = 1$

$\Rightarrow x = 0, \frac{x}{4} \Rightarrow \sec x = 1 \text{ or } \sqrt{2}$

$\therefore$ (R) $\to$ (2)

(S) $\cos\left(\sin^{-1}\sqrt{1 - x^2}\right) = \sin\left(\tan^{-1} x\sqrt{6}\right)$

$\Rightarrow \frac{x}{\sqrt{1 - x^2}} = \frac{x\sqrt{6}}{\sqrt{1 + 6x^2}} \Rightarrow x = \pm\frac{5}{2\sqrt{3}}$

$\therefore$ (S) $\to$ (1)

Hence (P) $\to$ (4), (Q) $\to$ (3), (R) $\to$ (2), (S) $\to$ (1)