PROBLEM

Prove that $\cos \tan^{-1} \sin \cot^{-1} x = \sqrt{\frac{x^2+1}{x^2+2}}$. ***(2002 - 5 Marks)***

SOLUTION

To prove that $\cos \tan^{-1} \sin \cot^{-1} x =$

L.H.S. $= \cos [\tan^{-1} (\sin (\cot^{-1} x))]$

$$= \cos [\tan^{-1} (\sin (\sin^{-1} \frac{1}{\sqrt{1+x^2}}))] \text{ if } x > 0$$

and $\cos [\tan^{-1} (\sin (\pi - \sin^{-1} \frac{1}{\sqrt{1+x^2}}))]$ if $x < 0$

In each case,

$$= \cos\left[\tan^{-1} \frac{1}{\sqrt{1+x^2}}\right] = \cos [\cos^{-}$$

= = *R.H.S.* *Hence Proved.*